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A NEW COSMOLOGICAL TEST FOR GENERAL RELATIVITY

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The effect of spatial variations of the Newton constant on the cosmic microwave background is studied. Constraints on the strong equivalence principle violation at the recombination time are then obtained with the help of WMAP data and of the standard theory of big-bang nucleosynthesis.

Новый космологический тест для общей теории относительности
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Исследуется влияние вариаций ньютоновской постоянной на космический микроволновой фон. Получены ограничения на нарушение сильного принципа эквивалентности в эпоху рекомбинации с помощью данных WMAP и стандартной теории космологического нуклеосинтеза.

1. Introduction

The latest results in cosmography as well as the latest observations of the cosmic microwave background (CMB) and of supernovae have reinforced the emergence of a canonical paradigm for cosmology. Most of the cosmological parameters constituting this concordance model are now known up to five per cent of relative accuracy. We can rely on these accurate values of the parameters if the fundamental hypotheses of the canonical paradigm are checked. Assumptions such as the cosmological principle [1–3] and the inflationary scenario [1, 4] have already been analyzed. But general relativity is poorly verified at cosmological scales and in the primordial ages. Deviations from general relativity predicted by the alternative theories of gravitation are not perceptible today [5, 6]. However, current experiments dedicated to cosmology could possibly detect signature of, e.g., the gravitational sector of string theory. Hence, our interest in testing general relativity in the early universe is to consolidate the confidence on the cosmological parameters as well as to probe or constrain new gravitational physics.

Previous works have already studied the impact of alternative theories of gravitation as pure [7] or extended [8–10] Brans-Dicke theory on the cosmic microwave background spectrum. Nevertheless, they do not consider breaking of one of the main features of general relativity: the strong equivalence principle (SEP). Only two metric theories of gravitation are based on this principle, namely, general relativity and Nordström's scalar theory. The latter is excluded by the observed light deflection by a gravitational potential. Consequently, if new gravitational physics exists beyond general relativity, a strong equivalence principle violation should be observed (while the Einstein and weak equivalence principles may be respected).

In this talk I summarize the results obtained in [11, 12]. The proposed test seeks a SEP violation in cosmological data for the cosmic microwave background with the help of the standard theory of big-bang nucleosynthesis. The aim of the work is to single out and to interpret SEP violation in the CMB power spectrum due to space variations of the Newtonian coupling.

2. Strong equivalence principle and its violation

The strong equivalence principle strengthens the Einstein equivalence principle in the following way. It extends the universality of free fall for test particles to compact bodies. Compact bodies are bodies with non-negligible self-binding gravitational energy. The SEP also enlarges the independence of non-gravitational experiment outcomes relative to the location in spacetime where the experiments are performed and relative to the speed of the freely falling frame in which they are run, to gravitational experiments. It follows that spacetime variations of the Newton *constant* G are sufficient to violate the SEP (but not the Einstein equivalence principle) since the results of gravitational experiments would then depend on the position in spacetime.

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2.1. The Nordtvedt effect

Variations of G induce changing the inertial mass of a given body through its self-binding gravitational energy. This effect can be implemented, in an effective way, by adding to the Lagrangian density for compact bodies a dependence on the new field $G(x)$. It ensues non-conservation of the energy-momentum tensor

$$\mathcal{L}_\mathrm{m} = \mathcal{L}_\mathrm{m}(g_{\mu\nu}, \Psi, G(x)) \quad \Rightarrow \quad T^{\mu\nu}{}_{|\nu} = \frac{\partial G}{\partial x_\mu}\frac{dT}{dG}, \quad (1)$$

where $g_{\mu\nu}$ is the metric and Ψ are non-gravitational fields. Since in this case the motion of a compact body is no more geodesic, the SEP is violated. The standard parameterization for space variations of G in the weak field limit is

$$G(\vec{x}) = G_\mathrm{N}\left[1 + \eta^\mathrm{gr}\frac{V(\vec{x})}{c^2}\right], \quad (2)$$

where $V(\vec{x})$ represents the external gravitational potential in which bodies are falling and G_N the value of the Newtonian coupling when V vanishes. The parameter η^gr characterizes the strong equivalence principle violation. This parameter equals zero for general relativity and the Nordström theory. The non-geodesic motion of massive bodies implies that the acceleration of these bodies in the potential V depends on their mass, so that the gravitational mass m^gr of a body differs from its inertial mass m^in. This is the so-called Nordtvedt effect [13, 14]:

$$m^\mathrm{gr} = m^\mathrm{in}(1 - \eta^\mathrm{gr}\ s). \quad (3)$$

The difference between m^gr and m^in originates from the sensitivity s of the mass to variations of G; s is also equal to the compactness of the considered body,

$$s = \frac{\partial \ln m^\mathrm{in}}{\partial \ln G} = \frac{|E^\mathrm{gr}|}{m^\mathrm{in}c^2}, \quad (4)$$

E^gr being its self-binding gravitational energy.

2.2. Today's constraints

Ranging the distance between the Earth and the Moon provides a constraint on the value of η^gr today[2] [5, 6]

$$\eta_0^\mathrm{gr} = (4.4 \pm 4.5)\ 10^{-4} \qquad \text{(LLR)}\ . \quad (5)$$

The second kind of test is based on orbital period decay observations of asymmetric and slightly relativistic binary systems and gives [15, 16]

$$\eta_0^\mathrm{gr} \leq 2.7\ 10^{-4} \qquad \text{(PSR J1141} - 6545). \quad (6)$$

The discrepancy between the above constraints and theoretical scenarios inspired from string theory is solved if the additional gravitational fields evolve as the Universe expands. In this case, the predicted value of η^gr, which should be about unity, appears as an initial condition maintained all over the radiation era. Hence, it is of primary importance to verify this prediction. In the following sections, we will look after constraints on the parameter η^gr at the time of recombination of electrons and protons contained in the primordial plasma.

3. Strong equivalence principle in cosmology

First, let us define our cosmological framework. We assume that the smooth expansion of a flat universe is governed by the cosmology of Friedmann and Lemaître. The primordial plasma contains baryons, cold dark matter, photons and three families of light neutrinos. Moreover, before recombination, we assume that baryons and photons are tightly coupled by Compton interactions and thus form a single fluid. Over the homogeneous Universe picture we add small adiabatic perturbations. The radiation pressure resistance to compression and rarefaction states of the plasma and falling of baryons in gravitational potentials generated by dark matter perturbations, trigger and maintain acoustic oscillations of the plasma. Temperature fluctuations of the plasma are related to photon density perturbations. These photons undergo a gravitational redshift at decoupling of baryons with photons when they climb out of the gravitational potentials. If we restrict ourselves to first-order perturbations, the plasma behaves like a set of independent oscillators with different spatial frequencies — the Fourier modes. This results in a succession of *acoustic peaks* in the CMB spectrum which represents the temperature anisotropy amplitude with respect to the inverse angular scale. In gravitational wells (hills), maximum compression (rarefaction) states at the decoupling time are associated with odd peaks. The weight of baryons shifts the oscillation zero point and appears in the CMB spectrum as an increase of the odd peak height. The relative height of the first and the second acoustic peaks are then related to the baryon density.

The issue is to compute the spectrum of photon perturbations at the recombination time in order to put a constraint on η_*^gr. The oscillation amplitude is proportional to the density of baryons. More exactly, we should use the gravitational density of the baryon-photon fluid, since acoustic oscillations originate in the gravitational interaction between the baryon-photon fluid and dark matter perturbations. But if the SEP is violated, the gravitational mass of the baryon fluid differs from the inertial one. If $\eta^\mathrm{gr} > 0$, the oscillation amplitude decreases since the weight of baryons is reduced relative to the inertial baryon density.

To quantify the effect of SEP violation on the CMB spectrum, we must evaluate the sensitivity of baryons to variations of G. For each Fourier mode we consider the falling of an homogeneous spherical body made of baryons with a radius equal to the wavelength of the

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[2] Quantities evaluated today or at the recombination of electrons and protons are indexed with a subscript $_0$ or $_*$, respectively.

Fourier mode. Therefore the baryon compactness related to the n-th CMB spectrum peak is

$$s_{b,*}^{(n)} \simeq 2.7\, \frac{\Omega_b}{n^2} \simeq \frac{0.1}{n^2}, \qquad (7)$$

where Ω_b is the ratio of the inertial density of baryons to the Einstein–de Sitter critical density. Solving Eq. (1) for the baryon-photon fluid, we find the following relation between the inertial density of baryons and their gravitational density $(\Omega_b)^{\rm gr}$ as measured through the height of the first and second acoustic peaks:

$$(\Omega_b)^{\rm gr} = (1 - \eta_*^{\rm gr} s_{b,*}^{(1)})\Omega_b. \qquad (8)$$

This determines a cosmological Nordtvedt effect, see [12] for more detail. Determining independently $(\Omega_b)^{\rm gr}$ and Ω_b gives a constraint on $\eta_*^{\rm gr}$.

4. Cosmological constraints

In this section, we derive two constraints on $\eta_*^{\rm gr}$. Firstly, we compare the gravitational density obtained by studying the impact of SEP violation on the acoustic peak height and the inertial density derived from other observables in the CMB spectrum. Secondly, we assume that the baryon density provided by CMB experiments is completely dominated by the gravitational density. The inertial density is then deduced from measurements of primordial light element abundances.

4.1. Cosmic microwave background

The density $(\Omega_b)^{\rm gr}$ is inferred from the height of the acoustic peaks. The inertial density can be found by analyzing the horizontal position of the acoustic peaks. Their position depends on the decoupling time and on the propagation speed of acoustic waves in the plasma. On the one hand, variation of the gravitational baryonic density due to the height uncertainty of the acoustic peaks measured by WMAP [17,18], and on the other hand, the permitted values of the inertial density deduced from the uncertainties about the horizontal localization of the first acoustic peak allow us to constrain $\eta_*^{\rm gr}$ [11]:

$$|\eta_*^{\rm gr}| \leq 0.6 \qquad (\text{CMB} - \text{CMB}). \qquad (9)$$

All limits and errors are given at 68% confidence level. Due to the approximations we made, this constraint should be taken as to an order of magnitude.

4.2. Big-bang nucleosynthesis

The second constraint on SEP violation comes from a comparison of CMB and big-bang nucleosynthesis (BBN) results. Since the density of baryons extracted from CMB experiments is mainly due to the gravitational density, we assume the density derived from WMAP, CBI and ACBAR experiments [17, 18] to be completely dominated by the gravitational density of baryons, $(\Omega_b)^{\rm gr}h^2 = 0.022 \pm 0.001$. Production of primordial light element abundances is not directly affected by SEP violation since BBN proceeds through non-gravitational interactions. Nonetheless, measurements of primordial abundances give direct constraints on alternative theories of gravitation (see e.g., [19]). Indeed, the production rates of light elements depend on the Hubble rate during nucleosynthesis, which is modified by new gravitational physics. We choose to compare the WMAP density with the density inferred from the relative abundance of deuterium and hydrogen D/H which is very sensitive to variations of the baryon density (see, e.g., [20]).

From the relative abundance D/H measured through quasar absorption lines, the baryon content of the Universe equals $\Omega_b h^2 = 0.0214 \pm 0.0020$ [21]. Finally, we derive a constraint on $\eta_*^{\rm gr}$ [11,12]:

$$\eta_*^{\rm gr} \simeq -0.3 \pm 1.0 \qquad \text{CMB} - \text{BBN}. \qquad (10)$$

5. Conclusion

We have proposed a cosmological test for general relativity. This test probes the strong equivalence principle at the recombination time through spatial variations of G. We have singled out and interpreted the impact of such a violation on the CMB power spectrum. Two constraints are proposed, one internal to the CMB, the second using CMB and BBN. No deviation from general relativity is found. Nevertheless, the constraints do not exclude the string-inspired value of $\eta^{\rm gr}$ which is about unity during the radiation era. To derive more confident bounds, we have to perform numerical simulations that go beyond the simple modification of baryon dynamics through Eq. (1) since the gravitational field equations are modified. We also have to run Monte-Carlo Markov chains to quantify the alteration of the cosmological parameter values. Hence, we have to work in a given alternative theory of gravitation with a specific running of $\eta^{\rm gr}$ and therefore of G in time, thus losing the generic property of our test.

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