

Searching for dominant rescattering sources in B to two pseudoscalar decays

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Abstract

Various rescattering sources are analyzed in the context of the SU(3) flavor symmetry. In particular, the possibility to account for intermediate charm at the hadronic level in B to PP is thoroughly investigated. Then, the rescattering sources are compared in light of recent B to two charmless pseudoscalar decay measurements, with emphasis on the size of strong phases and on patterns of direct CP-asymmetries.

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1 Introduction

Studies of CP violation in the B system offer a unique window into the intimate structure of the Standard Model, and possibly about New Physics. The dedicated experiments at Cornell, KEK and SLAC have begun to feed theorists with precise data to test the current understanding of CP violation, as well as alternate theories.

Recently, a number of the rare hadronic decays B to two charmless pseudoscalars have been measured (see [1-6]). These modes are interesting to constrain the weak angle γ of the unitary triangle. However, for this program to be achieved, theoretical control is needed over the strong final state interactions (FSI). In the present work, this problem is addressed phenomenologically, at the hadronic level (for other approaches, see for example [7]). The basic tools are flavor symmetries like isospin or SU(3) (see for example [8], [9], [10]) and Watson's theorem, or elasticity, for strong FSI (see [10], [11], [12]).

Up to now, a serious shortcoming of elasticity-based approaches was their inability to account for intermediate charm. In short-distance analyses, quark diagrams involving charm quarks do have non-negligible imaginary parts [13]. This led to the common belief that rescattering cannot be elastic. One purpose of the present paper is to show that it is possible within the hadronic framework of Watson's theorem elasticity to account for intermediate charm. Some results in that direction were already presented earlier [12]. Here, we will revert to the language of the usual quark diagram topologies. The theoretical foundations of our parametrization, which are articulated around a factorization property for FSI, are reviewed in the first part of the paper.

Another problem of elasticity-based approaches is that large strong phases are apparently required by current experimental data for B to two charmless pseudoscalars (see for example [14]). This is in disagreement with Regge computations [15], with the expectation for two particles flying apart with high momentum [16] and with QCD-factorization predictions [17]. In the second part of the paper, we will show that when intermediate charm is accounted for, the strong phases tend to be much smaller.

Finally, the last goal of the paper is to parametrize and quantify the corrections to the patterns for $B \rightarrow \pi K$ direct CP-asymmetries presented in [12]. These patterns were designed to discriminate among dominant rescattering sources, and we will here show that this capability is not altered.

2 Theoretical Framework

The starting point of our analysis is the "generalized Watson Theorem" (for details, see [10], [11], [12]), which permits the factorization of weak decay amplitudes into a factor invariant under CP and a factor that gets complex conjugated under CP

$$W = \sqrt{S}W_b \tag{1a}$$

$$CP(W) = \sqrt{S}W_b^* \tag{1b}$$

where W is a vector containing the full decay amplitudes. Consequently, the real (up to CKM factors) W_b is identified as the *bare decay amplitude* (i.e. before FSI) denoted as

$$W = \begin{pmatrix} B \rightarrow X_1 \\ \vdots \end{pmatrix}, \quad W_b = \begin{pmatrix} B \rightarrow \{X_1\} \\ \vdots \end{pmatrix} \quad (2)$$

The unitary *rescattering matrix* \sqrt{S} contains all the CP-conserving strong phases and describes final state interactions (FSI). In general, one defines the rescattering *eigenchannels* C_i

$$\begin{pmatrix} C_1 \\ \vdots \end{pmatrix} = O_C \begin{pmatrix} \{X_1\} \\ \vdots \end{pmatrix} \quad (3)$$

as the basis in which \sqrt{S} is diagonal

$$S_{diag} = O_C \cdot S \cdot O_C^t = \begin{pmatrix} e^{2i\delta_{C_1}} & 0 & \dots \\ 0 & e^{2i\delta_{C_2}} & \dots \\ \vdots & \vdots & \ddots \end{pmatrix} \quad (4)$$

with strong phases (or rescattering *eigenphases*) δ_{C_i} as diagonal elements. *Elasticity* (sometimes referred to as "quasi-elasticity") is defined as the conservation of the total decay probability, which follows from the unitarity of \sqrt{S}

$$\|W\|^2 = \|\sqrt{S}W_b\|^2 = \|W_b\|^2 \quad (5)$$

Assuming that CP and CPT hold for the strong interactions generating \sqrt{S} implies that O_C is orthogonal (up to some phase conventions) since then \sqrt{S} is unitary and symmetric. The angles parametrizing O_C are then called *mixing angles* since they govern the FSI mixings among decay channels.

In principle, \sqrt{S} should be a $n \times n$ matrix with n the number of B -meson decay channels which can be connected by strong interactions. In practice, however, we expect that to a good approximation this matrix is bloc-diagonal. Unfortunately, it is not known at present how large the extent of each bloc should be to capture the essential rescattering physics. In the present work, we assume that two-body \rightleftharpoons n -body rescatterings are negligible (as could be justified from $1/N$ arguments [18]), or at least suppressed by large cancellations.

The most restrictive (though non-trivial) approximation is that of SU(2)-elasticity: each bloc of \sqrt{S} corresponds to an isospin multiplet, which means that only rescatterings like $\pi\pi \rightleftharpoons \pi\pi$ or $K\bar{K} \rightleftharpoons K\bar{K}$ (see [12]) are allowed. A more flexible approach is to consider SU(3)-elasticity, to open rescatterings between different iso-multiplets like $\pi\pi \rightleftharpoons K\bar{K}$ or $\pi K \rightleftharpoons \eta_8 K$. In an attempt to capture all the relevant physics, rescattering channels like $D\bar{D} \rightleftharpoons \pi\pi$ or $D_s\bar{D} \rightleftharpoons \pi K$ should also be opened to account for intermediate charm. However, sizeable $D\bar{D} \rightleftharpoons PP$ rescatterings like those implied by SU(4)-elasticity would average decay amplitudes and produce $Br(B \rightarrow D\bar{D}) \sim Br(B \rightarrow PP)$, in clear disagreement with experiment

$$Br^{\text{exp}}(B \rightarrow D_s\bar{D}) \sim 10^{-2} \gg Br^{\text{exp}}(B \rightarrow K\pi) \sim 10^{-5} \quad (6)$$

Intermediate charm rescattering effects are thus quite small, and can be accounted for as distortions of SU(3)-elasticity. We call enlarged SU(3)-elasticity this parametrization of FSI, since the SU(3) flavor symmetry remains exact. Conversely, even if all the rescatterings (both two-body and multi-body) are small, the $D\bar{D} \rightleftharpoons PP$ ones may still lead to sizeable effects because of Eq.(6), so it is desirable to have an adequate formalism at hand.

By definition, bare amplitudes W_b are real (except for CKM matrix elements) since they do not contain any FSI effects (see Eqs.(1)). To parametrize them, and to make contact with the short-distance weak decay of the b quark, it is convenient to introduce quark diagrams which account for the possible flavor flows (see, for example, [8], [9], [10]). To first order in the weak interactions, and to all orders in the strong interactions (except FSI), the six topologies needed for $B \rightarrow \{PP\}$, $P = \pi, K, \eta_8$ and $B \rightarrow \{D\bar{D}\}$, $D = D, D_s$ are shown in Fig.1.

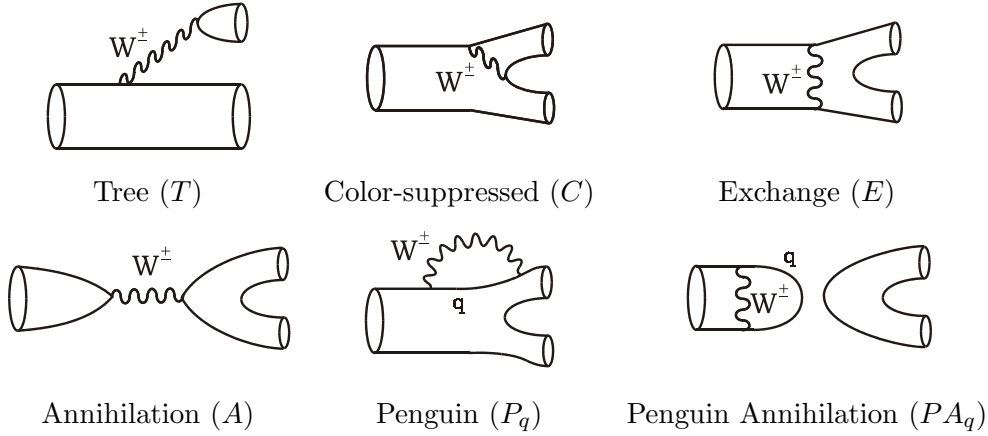


Figure 1: Quark diagrams for non-singlet final states, $q = u, c, t$.

In principle, second-order weak interaction topologies are very small, except for those depicted in Fig.2 involving an enhancement factor m_{top}^2/M_Z^2 .

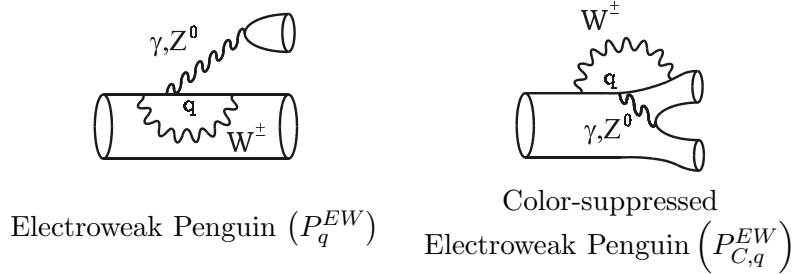


Figure 2: Second-order weak interaction quark diagrams.

The SU(3) symmetry is implemented at the level of bare decay amplitudes by identifying quark diagrams that differ only by the interchange $u \leftrightarrow d \leftrightarrow s$. The badly broken SU(4)

symmetry is not invoked and, consequently, quark diagrams for $B \rightarrow \{D\bar{D}\}$ will be distinguished from $B \rightarrow \{PP\}$ ones by a subscript D .

Rescattering matrices have an interesting property of factorization. If each of the strong phases is expressed as the sum of two parts

$$\delta_{C_i} = \delta_{C_i}^a + \delta_{C_i}^b, \quad (7)$$

then, from the unitarity of \sqrt{S} (see Eq.(4))

$$\sqrt{S}(\delta_{C_1}, \dots) = \sqrt{S}(\delta_{C_1}^a, \dots) \cdot \sqrt{S}(\delta_{C_1}^b, \dots) \quad (8)$$

Now, we can define effective "bare" amplitudes containing the $\delta_{C_i}^b$ part of the CP-conserving strong phases:

$$W = \sqrt{S}(\delta_{C_1}, \dots) \cdot W_b \equiv \sqrt{S}(\delta_{C_1}^a, \dots) \cdot W_b^{eff} \quad (9)$$

In just the same way W_b is parametrized in terms of real bare quark diagrams T, C, \dots , the effective bare amplitudes W_b^{eff} are decomposed into complex effective quark diagram amplitudes T^{eff}, C^{eff}, \dots (i.e. $W_b^{eff} = W_b(T \rightarrow T^{eff}, C \rightarrow C^{eff}, \dots)$). It is important to realize that in the literature, quark diagrams are often introduced at the level of W_b^{eff} . One should then be very careful in understanding which part of the FSI they contain, so that different approaches can be related. Finally, note that $CP(W_b^{eff}) \neq (W_b^{eff})^*$ since some CP-invariant strong phases are now included into them (compare with Eqs.(1)).

2.1 SU(3)-elastic Rescattering

SU(3)-elasticity is enforced by taking definite SU(3) states as rescattering eigenchannels. Taking as an example the set of decay amplitudes

$$W = \begin{pmatrix} B^+ \rightarrow K^+ \eta_8 \\ B^+ \rightarrow K^+ \pi^0 \\ B^+ \rightarrow K^0 \pi^+ \\ B^+ \rightarrow D_s^+ \bar{D}^0 \end{pmatrix}, \quad (10)$$

the rescattering matrix S is diagonal

$$S_{diag} = \text{diag} \left(e^{2i\delta_{27}}, e^{2i\delta_{27}}, e^{2i\delta_8}, e^{2i\delta_8^D} \right) \quad (11)$$

in the basis defined by

$$\begin{pmatrix} |27, 3/2, 1/2, 1\rangle \\ |27, 1/2, 1/2, 1\rangle \\ |8, 1/2, 1/2, 1\rangle \\ |8_D, 1/2, 1/2, 1\rangle \end{pmatrix} = \begin{pmatrix} 0 & -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{3}{\sqrt{10}} & -\frac{1}{\sqrt{30}} & -\frac{1}{\sqrt{15}} & 0 \\ \frac{1}{\sqrt{10}} & -\sqrt{\frac{3}{10}} & -\sqrt{\frac{3}{5}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \{K^+ \eta_8\} \\ \{K^+ \pi^0\} \\ \{K^0 \pi^+\} \\ \{D_s^+ \bar{D}^0\} \end{pmatrix} \quad (12)$$

Therefore, the SU(3)-elastic rescattering matrix is

$$\sqrt{S_{SU(3)}} = \begin{pmatrix} \frac{9e^{i\delta_{27}} + e^{i\delta_8}}{10} & \frac{\sqrt{3}(e^{i\delta_{27}} - e^{i\delta_8})}{10} & \frac{\sqrt{3}(e^{i\delta_{27}} - e^{i\delta_8})}{\sqrt{50}} & 0 \\ \frac{\sqrt{3}(e^{i\delta_{27}} - e^{i\delta_8})}{10} & \frac{7e^{i\delta_{27}} + 3e^{i\delta_8}}{10} & \frac{3(e^{i\delta_8} - e^{i\delta_{27}})}{\sqrt{50}} & 0 \\ \frac{\sqrt{3}(e^{i\delta_{27}} - e^{i\delta_8})}{\sqrt{50}} & \frac{3(e^{i\delta_8} - e^{i\delta_{27}})}{\sqrt{50}} & \frac{2e^{i\delta_{27}} + 3e^{i\delta_8}}{5} & 0 \\ 0 & 0 & 0 & e^{i\delta_8^D} \end{pmatrix} \quad (13)$$

See how SU(3)-elasticity fixes both the eigenphases Eq.(11) *and* the eigenchannels Eq.(12): the $\{D_s^+ \bar{D}^0\}$ and $\{PP\}$ channels are not coupled and $\sqrt{S_{SU(3)}}$ has a characteristic block-diagonal form. Other sets of states are treated similarly (see [10]).

Let us now concentrate on the $PP \rightleftharpoons PP$ part of $\sqrt{S_{SU(3)}}$. If each strong phase is expressed as $\delta_i = \delta_i^l + \delta_i^s$ with $i = 1, 8, 27$ (one could think of the FSI short and long-distance parts), we can define effective, complex quark diagrams by absorbing the δ_i^s part of the rescattering

$$W_b^{eff} = \sqrt{S_{SU(3)}} (\delta_{27}^s, \delta_8^s, \delta_1^s) W_b \quad (14)$$

Explicitly, effective quark diagrams can be expressed in terms of bare (real) ones as (omitting electroweak penguins for now)

$$\text{SU(3)-elasticity} \begin{cases} X^{eff} = X e^{i\delta_8^s} + \frac{e^{i\delta_{27}^s} - e^{i\delta_8^s}}{2} (T + C) & X = T, C \\ Y^{eff} = Y e^{i\delta_8^s} - \frac{e^{i\delta_{27}^s} - e^{i\delta_8^s}}{10} (T + C) & Y = E, A, P_u \\ P_{c,t}^{eff} = P_{c,t} e^{i\delta_8^s} \\ PA_u^{eff} = PA_u e^{i\delta_1^s} - \frac{e^{i\delta_{27}^s} - e^{i\delta_8^s}}{20} (T + C) + \frac{e^{i\delta_1^s} - e^{i\delta_8^s}}{12} (3T - C + 8E + 8P_u) \\ PA_{c,t}^{eff} = PA_{c,t} e^{i\delta_1^s} + \frac{2(e^{i\delta_1^s} - e^{i\delta_8^s})}{3} P_{c,t} \end{cases} \quad (15)$$

The remaining δ_i^l part of the rescattering is accounted for by acting with $\sqrt{S_{SU(3)}} (\delta_{27}^l, \delta_8^l, \delta_1^l)$ on W_b^{eff} .

In the following, we shall not use Eqs.(15). What we want to point out by writing them is that one should be careful when dealing with effective quark diagrams since obviously, the rough scalings (penguins are discussed in the next section)

$$\begin{aligned} & 1 : T \\ & \mathcal{O}(\lambda) : C \\ & \mathcal{O}(\lambda^2) : E, A \end{aligned} \quad (16)$$

with $\lambda \approx 1/5$ [9] may no longer be valid. For example, for the $B^+ \rightarrow K^0 \pi^+$ mode, while A is helicity-suppressed and can be safely ignored, A^{eff} receives unsuppressed contributions from T (provided $\delta_{27}^s - \delta_8^s$ is non-negligible), as already emphasized in [19]. Similarly, for $B^0 \rightarrow \pi^+ \pi^-$, E and PA can be neglected, but E^{eff} and PA^{eff} may not. The Eqs.(15) illustrate in the specific SU(3)-elastic case a more general fact: if one starts with a parametrization in terms of effective quark diagrams without specifying anything about

FSI, all the effective quark diagrams are then arbitrary complex numbers and this costs much in terms of free parameters (except, of course, if one has a definite computation scheme for effective quark diagrams).

2.2 Intermediate Charm Rescattering

We now open the rescattering channel $D\bar{D} \rightleftharpoons PP$ [20]. For the set of states Eq.(10), general rescattering eigenchannels are defined by mixing 8 with 8_D (for a discussion on the formalism used here, see [10], [12]):

$$\begin{pmatrix} C_1(27, 3/2) \\ C_2(27, 1/2) \\ C_3(8, 1/2) \\ C_4(8, 1/2) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \chi_8 & \sin \chi_8 \\ 0 & 0 & -\sin \chi_8 & \cos \chi_8 \end{pmatrix} \begin{pmatrix} |27, 3/2, 1/2, 1\rangle \\ |27, 1/2, 1/2, 1\rangle \\ |8, 1/2, 1/2, 1\rangle \\ |8_D, 1/2, 1/2, 1\rangle \end{pmatrix} \quad (17)$$

The rescattering matrix in the C_i basis is

$$S_{diag} = \text{diag} \left(e^{2i\delta_{C_1}}, e^{2i\delta_{C_2}}, e^{2i\delta_{C_3}}, e^{2i\delta_{C_4}} \right) \approx \text{diag} \left(e^{2i\delta_{27}}, e^{2i\delta_{27}}, e^{2i\delta_8}, e^{2i\delta_8^D} \right) \quad (18)$$

since the mixing angle χ_8 should be small (see Eq.(6)). In the physical state basis, we then find

$$\begin{aligned} \sqrt{S_{\chi_8}} &= \sqrt{S_{SU(3)}} + \sin(2\chi_8) \frac{e^{i\delta_8} - e^{i\delta_8^D}}{2\sqrt{10}} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -\sqrt{3} \\ 0 & 0 & 0 & -\sqrt{6} \\ 1 & -\sqrt{3} & -\sqrt{6} & 0 \end{pmatrix} \\ &+ \sin^2(\chi_8) \frac{e^{i\delta_8} - e^{i\delta_8^D}}{10} \begin{pmatrix} -1 & \sqrt{3} & \sqrt{6} & 0 \\ \sqrt{3} & -3 & -3\sqrt{2} & 0 \\ \sqrt{6} & -3\sqrt{2} & -6 & 0 \\ 0 & 0 & 0 & 10 \end{pmatrix} \end{aligned} \quad (19)$$

with $\sqrt{S_{SU(3)}}$ given in Eq.(13). In physical terms, one can understand χ_8 as a $D\bar{D}$ - PP "coupling constant": the $\sin(2\chi_8)$ term describes the $D\bar{D}$ pollution of PP states (which proceeds through $D\bar{D} \xrightarrow{\chi_8} PP, D\bar{D} \xrightarrow{\chi_8} PP \xrightarrow{\chi_8} D\bar{D} \xrightarrow{\chi_8} PP$, etc) while the $\sin^2(\chi_8)$ terms account for "inelastic" distortion of the rescattering among $\{PP\}$ states due to the exchange of probability with the $D\bar{D}$ channel ($PP \xrightarrow{\chi_8} D\bar{D} \xrightarrow{\chi_8} PP$, etc). The common factor $e^{i\delta_8} - e^{i\delta_8^D}$ acts as a kinematical suppression since it decreases as the relative momentum between decay products increases ($\delta_8 = \delta_8^D = 0$ in the limit $M_B \rightarrow \infty$ [16]). Finally, note well that SU(3) remains exact, and that the rescattering is elastic (conserved total probability) for the full set of states $\{PP, D\bar{D}\}$ since $\sqrt{S_{\chi_8}}$ is unitary for all χ_8 .

Other set of states are treated similarly (the mixings among $Y = 0, T_3 = 0$ states is described in Appendix A). All in all, six parameters are needed to describe all the rescattering mixings (one of the strong phase can be eliminated)

$$\begin{aligned} PP : (8 \otimes 8)_S &= 27 \oplus 8 \oplus 1 \rightarrow \delta_{27}, \delta_8, \delta_1 \\ &\quad \uparrow \chi_8 \quad \uparrow \chi_1 \\ D\bar{D} : 3 \otimes \bar{3} &= 8_D \oplus 1_D \rightarrow \delta_8^D, \delta_1^D \end{aligned} \quad (20)$$

We expect that $\chi_8, \chi_1 \ll 1$ since $D\bar{D} \rightarrow PP$ proceeds through the annihilation of the $c\bar{c}$ pair into light quarks, and $\delta_{27}, \delta_8, \delta_1 < \delta_8^D, \delta_1^D$ since there is less phase-space for $D\bar{D}$ than for PP . As said before, SU(4)-elasticity is not implemented: it would fix the χ_i to some large values and introduce relations between $\delta_{27}, \delta_8, \delta_1$ and δ_8^D, δ_1^D , leading to $Br(B \rightarrow PP) \approx Br(B \rightarrow D\bar{D})$ (see Eq.(6)). Let us stress again that SU(4) is used at no stage of the present analysis.

The factorization property of \sqrt{S} can now be used to separate the $D\bar{D} \rightleftharpoons PP$ rescattering effects from the SU(3)-elastic part. For the set of states Eq.(10), this is achieved by splitting the phases as (see Eq.(19))

$$\sqrt{S_{\chi_8}}(\delta_{27}, \delta_8, \delta_8^D) = \underbrace{\sqrt{S_{\chi_8}}(\delta_{27}, \delta_8, \delta_8)}_{\substack{PP \rightleftharpoons PP \\ D\bar{D} \rightleftharpoons D\bar{D}}} \cdot \underbrace{\sqrt{S_{\chi_8}}(0, 0, \delta_8^D - \delta_8)}_{D\bar{D} \rightleftharpoons PP} \quad (21)$$

The first factor does not depend on χ_8 and is related to the SU(3)-elastic rescattering matrix Eq.(13) as $\sqrt{S_{\chi_8}}(\delta_{27}, \delta_8, \delta_8) = \sqrt{S_{SU(3)}}(\delta_{27}, \delta_8, \delta_8)$, while the second factor contains all the effects of $D\bar{D} \rightleftharpoons PP$ mixing and becomes trivial if either $\delta_8^D = \delta_8$ or $\chi_8 = 0$. Proceeding similarly with the other multiplets, we now absorb the $D\bar{D} \rightleftharpoons PP$ part of the rescattering into effective quark diagrams

$$\text{On-shell } c\bar{c} \text{ FSI} \begin{cases} P_c^{eff} = P_c - \beta \\ PA_c^{eff} = PA_c + \frac{2}{3}\beta + \alpha \end{cases} \quad (22)$$

with

$$\alpha = \frac{1}{2\sqrt{3}} \left(1 - e^{i(\delta_1^D - \delta_1)}\right) \chi_1 T_D, \quad \beta = \sqrt{\frac{3}{5}} \left(1 - e^{i(\delta_8^D - \delta_8)}\right) \chi_8 T_D \quad (23)$$

To reach these expressions, we have retained only the dominant T_D contribution to $B \rightarrow D\bar{D}$ decay amplitudes (see Fig.1) and the first order in χ_i (so that $PP \rightarrow D\bar{D} \rightarrow \dots \rightarrow PP$ effects are neglected, see Eq.(19)). Importantly, all the other quark diagrams are unaffected by $D\bar{D} \rightleftharpoons PP$. Since the P_c and PA_c quark diagrams are precisely the ones involving the charm quark, we interpret Eq.(22) as a hadronic representation for on-shell intermediate $c\bar{c}$ quarks, as previously mentioned in [12] (see Fig.3).

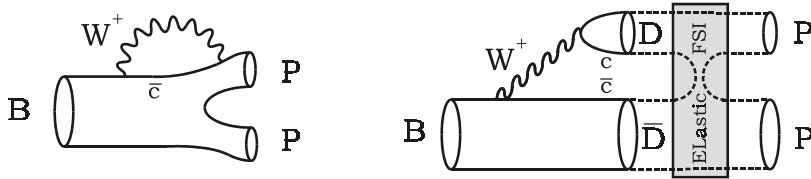


Figure 3: The standard quark-level penguin amplitude P_c and the hadronic-level penguin-like $B \rightarrow \{D\bar{D}\} \rightarrow PP$ contribution.

In general, because of renormalization of the penguin loop, it is desirable to use the unitarity of the CKM matrix to write ($\lambda_q \equiv V_{qb}^* V_{qd(s)}$)

$$\lambda_u P_u + \lambda_c P_c + \lambda_t P_t = \lambda_u (P_u - P_c) + \lambda_t (P_t - P_c) \equiv \lambda_u (P_{u-c}) + \lambda_t (P_{t-c}) \quad (24)$$

At the B -mass scale, we expect that P_{u-c} is small since $m_u, m_c \ll m_b$. Interestingly, when on-shell intermediate $c\bar{c}$ is treated at the hadronic level, the P_c short-distance absorptive part has to be discarded and to a large extent the cancellation between P_u and P_c is preserved. Therefore, one can expect the rough scaling of the dominant penguin contributions [9]

$$\begin{aligned}
& 1 : T \\
& \mathcal{O}(\lambda) : P_{t-c} \\
& \mathcal{O}(\lambda^2) : P_{t-c}^{EW} \\
& \mathcal{O}(\lambda^3) : P_{C,t-c}^{EW}, PA_{t-c}
\end{aligned} \tag{25}$$

and $|X_{u-c}| < |X_{t-c}|$ with $X = P, PA, P^{EW}, P_C^{EW}$ to be valid. In the following, X_{u-c} will be neglected and the effects of intermediate charm will be accounted for by defining

$$\text{On-shell } c\bar{c} \text{ FSI } \begin{cases} \lambda_t P_{t-c}^{eff} = \lambda_t P_{t-c} - \lambda_c \beta \\ \lambda_t PA_{t-c}^{eff} = \lambda_t PA_{t-c} + \lambda_c \left(\frac{2}{3}\beta + \alpha\right) \end{cases} \tag{26}$$

Note well that P_{t-c}^{eff} and PA_{t-c}^{eff} involve a combination of CP-conserving and CP-violating phases.

Our method for implementing intermediate on-shell charm has semi-inclusive features. To see this, note first that all the developments above can be repeated replacing $D\bar{D}$ by $D^*\bar{D}^*, D^{**}\bar{D}^{**}, \dots$. Now, taking into account the effects on $B \rightarrow PP$ of rescatterings from all the charmed meson states amounts to the definitions Eqs.(22) or Eqs.(26) with α, β replaced by

$$\tilde{\alpha} = \frac{1}{2\sqrt{3}}F_1, \quad \tilde{\beta} = \sqrt{\frac{3}{5}}F_8, \quad F_k = \sum_{i=D, D^*, \dots} \left(1 - e^{i(\delta_k^i - \delta_k)}\right) \chi_k^i T_i \tag{27}$$

and this holds even if $D\bar{D} \rightleftharpoons D^*\bar{D}^*, \dots$ rescatterings are present thanks to the factorization property Eq.(8) or Eq.(21). Finally, since in any case, the complex numbers $\tilde{\alpha}$ and $\tilde{\beta}$ can be parametrized as in Eq.(23), this change is irrelevant (except, of course, that χ and T_D can no longer be determined separately). In other words, as long as the χ_i are small (i.e. when the last term of Eq.(19) can be dropped), Watson's theorem implies that no matter the precise physics in the charmed sector, its impact on $B \rightarrow PP$ will always amount to a redefinition of the penguin amplitudes P_c and PA_c (or P_{t-c} and PA_{t-c}). In the following, for simplicity, we assume that $D\bar{D} \rightleftharpoons PP$ dominates so as to give estimations of χ_i . Finally, it is also straightforward to apply the formalism to charmonium modes $P\eta_c$ and/or $\eta_0\eta_c$ (see [21]), simply by substituting T_D by the C_{η_c} quark diagram contributing to $P\eta_c$ in β , and T_D by the C'_{η_c} contributing to $\eta_0\eta_c$ in α .

Let us conclude this section by a more technical comment. If we note $\bar{3}_c$ the $\bar{b}c.\bar{c}d$ or $\bar{b}c.\bar{c}s$ part of the weak Hamiltonian (see [10]), P_c, PA_c and the definitions Eq.(22) are expressed in terms of SU(3) reduced matrix elements as

$$\begin{cases} P_c = -\langle 8|\bar{3}_c|3\rangle \\ PA_c = \frac{2}{3}\langle 8|\bar{3}_c|3\rangle + \langle 1|\bar{3}_c|3\rangle \end{cases} \rightarrow \begin{cases} \langle 8|\bar{3}_c|3\rangle^{eff} = \langle 8|\bar{3}_c|3\rangle + \beta \\ \langle 1|\bar{3}_c|3\rangle^{eff} = \langle 1|\bar{3}_c|3\rangle + \alpha \end{cases} \tag{28}$$

As expected, the $\alpha(\beta)$ term involving χ_1 (χ_8) contributes only to the reduced matrix element involving the 1 (8) final state, respectively.

2.3 Final Parametrization

All the pieces can now be put together to construct the final parametrizations of $B \rightarrow PP$ decay amplitudes assuming enlarged SU(3)-elasticity. From the previous sections:

- 1- All $PP \rightleftharpoons PP$ rescattering effects should be contained in \sqrt{S} , to preserve the scalings between quark diagram amplitudes, and will be treated assuming exact SU(3) (see Eqs.(15) and Eqs.(16)).
- 2- Intermediate charm can be treated at the hadronic level as $D\bar{D} \rightleftharpoons PP$ rescattering effects, and absorbed into P_{t-c} and PA_{t-c} (see Eq.(26)).

The first point is important since it allows us to reduce the number of free parameters. Indeed, combining Eqs.(16) and Eqs.(25) with CKM coefficients, it appears as sufficient to consider only four topologies ($\lambda_q^{q'} \equiv V_{qb}^* V_{qq'}$)

$$\begin{aligned} \Delta S = 0 : \quad & \lambda_u^d T, \quad \lambda_t^d P_{t-c}, \lambda_u^d C \\ \Delta S = 1 : \quad & \lambda_t^s P_{t-c}, \quad \lambda_u^s T, \lambda_t^s P_{t-c}^{EW} \end{aligned} \quad (29)$$

since in any case, SU(3) breaking effects in the dominant T and P_{t-c} contributions should be greater than $\mathcal{O}(\lambda^2) \sim 4\%$.

Once PA is neglected, it seems justified to set also (see Eq.(26))

$$\frac{2}{3}\beta + \alpha = 0 \Leftrightarrow \begin{cases} \delta_1^D - \delta_1 = \delta_8^D - \delta_8 \\ \chi \equiv \chi_8 = -\frac{\sqrt{5}}{4}\chi_1 \end{cases} \quad (30)$$

since this combination describes rescatterings that proceeds through the vacuum (Eq.(30) may seem to imply a fine tuning between α and β , but this is equivalent to the usual assumption that PA_c is small, see Eqs.(28)).

There remain eight free parameters

$$\begin{aligned} \text{QD amplitudes: } & T, C, P_{t-c}, P_{t-c}^{EW} \\ \text{SU(3) rescatterings: } & \delta_{27} - \delta_1, \delta_8 - \delta_1 \\ \text{On-shell } c\bar{c} \text{ FSI: } & \chi T_D, \delta_8^D - \delta_1 \end{aligned} \quad (31)$$

The expressions for decay amplitudes are collected in Appendix B, in which the definition Eq.(26) is used, δ_1 is set to zero and CKM coefficients are omitted. Also, the color-allowed electroweak penguin has to be introduced as $C \rightarrow C + P_{t-c}^{EW}$ [9]. Decay amplitudes for $B \rightarrow D\bar{D}$, assuming that $D\bar{D}, PP \rightleftharpoons D^*\bar{D}^*, \dots$ are negligible, are also given. In that case, measurements of $B \rightarrow D\bar{D}$ branchings can serve to determine both T_D and the strong phase $\delta_8 - \delta_1$ of $PP \rightleftharpoons PP$ rescattering. This fact originates in the coherence requirement Eq.(30), which implies (provided $\chi \neq 0$)

$$\frac{Br(B^0 \rightarrow D^+ D^-) \Gamma(B^0)}{Br(B^+ \rightarrow D^+ \bar{D}^0) \Gamma(B^+)} = \frac{5 + 4 \cos(\delta_8^D - \delta_1^D)}{9} = \frac{5 + 4 \cos(\delta_8 - \delta_1)}{9} \quad (32)$$

3 Numerical Examples

For our numerical examples, we consider only one source of rescattering: either SU(3)-elastic ones ($PP \rightleftharpoons PP$) or on-shell intermediate charm ($D\bar{D} \rightleftharpoons PP$). The sets of free parameters in each case are

$$\begin{array}{l}
 \text{Amplitudes:} \quad T, P_{t-c}, C, P_{t-c}^{EW} \\
 \text{Rescattering:} \quad \begin{array}{cc}
 \swarrow & \searrow \\
 PP \rightleftharpoons PP & D\bar{D} \rightleftharpoons PP \\
 \delta_{27} - \delta_1, \delta_8 - \delta_1 & \chi T_D, \delta_8^D - \delta_1
 \end{array}
 \end{array} \quad (33)$$

with, in addition, the weak angle γ . To leading order, one can further neglect C and P_{t-c}^{EW} . Though the formalism allows for general analyses, the precision of the experimental data is not yet sufficient for meaningful combined analyses.

For each fit, we give the values of the parameters in the text, while the corresponding values for branchings (Table V), A_{CP} (Table VI) and $S_{f\bar{f}}$ (Table VII) are in Appendix C, along with the details of our fitting procedure.

3.1 SU(3)-elasticity (no $D\bar{D} \rightleftharpoons PP$)

For the first example, we take the T and P_{t-c} amplitudes and $\delta_8 - \delta_1$, $\delta_{27} - \delta_1$ strong phases (so we set $\chi = 0$). For various (fixed) values of the weak angle γ we find

γ	60°	80°	100°
χ_{\min}^2	18.3	16.3	17.7
T	0.65	0.70	0.76
P_{t-c}	0.11	0.11	0.10
$\delta_8 - \delta_1$	47°	52°	62°
$\delta_{27} - \delta_1$	100°	98°	101°

Table I: T, P_{t-c} , SU(3)-elasticity.

Quark diagram amplitudes given in the table are dimensionless and produce B^0 branchings.

Under this SU(3)-elastic parametrization, the $B \rightarrow \pi K$ branchings are dominated by the P_{t-c} penguin amplitude and verify

$$Br(\pi^0 K^+) : Br(\pi^+ K^0) : Br(\pi^0 K^0) : Br(\pi^- K^+) \approx 1/2 : 1 : 1/2 : 1 \quad (34)$$

While the pattern of direct CP-asymmetries is (see [12])

$$A_{\pi^0 K^+} : A_{\pi^+ K^0} : A_{\pi^0 K^0} : A_{\pi^- K^+} \approx 2 : -\frac{1}{2} : -\frac{3}{2} : 1 \quad (35)$$

no matter the strong phases.

As already found in [14], if the current CP-asymmetry measurement central values are to be trusted, they require large $PP \rightleftharpoons PP$ phases. Then, in the $B \rightarrow \pi\pi$ sector,

dominated by T , large rescatterings occur and the $\pi^0\pi^0$ mode is fed from $B \rightarrow \{\pi^+\pi^-\}$. The resulting pattern is close to

$$Br(\pi^+\pi^0) : Br(\pi^+\pi^-) : Br(\pi^0\pi^0) \approx 1 : 1 : 1/2 \quad (36)$$

Typically, $Br(B^0 \rightarrow \pi^0\pi^0) > 1.5 \times 10^{-6}$ under SU(3)-elasticity. For $\pi^+\pi^-$ time-dependent asymmetry, both $A_{\pi^+\pi^-}$ and $S_{\pi^+\pi^-}$ are found compatible with current experimental data (see appendix C):

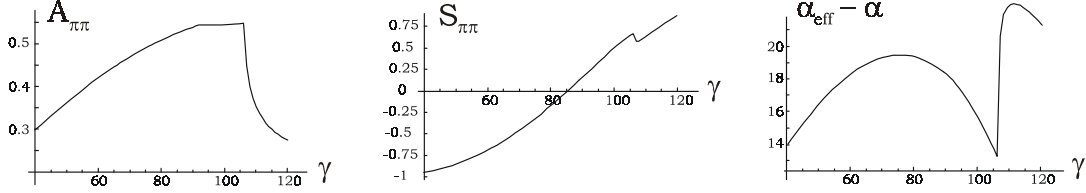


Figure 4: Fit result for $A_{\pi^+\pi^-}$, $S_{\pi^+\pi^-}$ and α_{eff} under SU(3)-elasticity.

$A_{\pi^+\pi^-}$ is not very sensitive to γ and stays in the range 0.3 to 0.5. On the other hand, $S_{\pi^+\pi^-}$ varies much, and it appears that the present Belle measurement exclude $\gamma > 90^\circ$ to 3σ . However, the dependence of $S_{\pi^+\pi^-}$ on γ is rather trivial. Defining the parameter α_{eff} as

$$S_{\pi^+\pi^-} = \sqrt{1 - A_{\pi^+\pi^-}^2} \sin(2\alpha_{eff}) \quad (37)$$

such that $\alpha_{eff} = \alpha \equiv \pi - \beta - \gamma$ if only T contributes to $B^0 \rightarrow \pi^+\pi^-$, the last graph shows that the deviation $\alpha_{eff} - \alpha$ stays small compared to γ .

Finally, for $B \rightarrow K\bar{K}$, no clear pattern emerges. However, one can note that because of large rescatterings from $B \rightarrow \{\pi^+\pi^-\}$, the $B^+ \rightarrow K^+K^-$ branching is saturating its present experimental upper bound. Also, the $B^+ \rightarrow K^+\bar{K}^0$ and $B^0 \rightarrow K^0\bar{K}^0$ direct CP-asymmetries tend to be sizeable, between -1 and -0.5 .

Just for illustration, if we now introduce the subleading C and P_{t-c}^{EW} topologies, the best-fit values are

γ	60°	80°	100°
χ_{\min}^2	10.1	10.5	15.4
T	0.60	0.66	0.67
C	0.21	0.17	0.17
P_{t-c}	0.11	0.11	0.10
P_{t-c}^{EW}	0.011	0.008	0.005
$\delta_8 - \delta_1$	53°	57°	32°
$\delta_{27} - \delta_1$	88°	89°	59°

Table II: $T, P_{t-c}, C, P_{t-c}^{EW}$, SU(3)-elasticity.

The most noticeable feature is a reduction of the strong phases, at the cost of a quite large C amplitude, and a significant decrease in χ_{\min}^2 .

To within 20%, the introduction of the additional quark diagram amplitudes does not modify the patterns described above. For example, the ratios of CP-asymmetries in the πK sector are now

$$A_{\pi^0 K^+} : A_{\pi^+ K^0} : A_{\pi^0 K^0} : A_{\pi^- K^+} = 2 + \frac{8}{5}\kappa_{-1} : -\frac{1}{2} + \frac{3}{5}\kappa_{2/3} : -\frac{3}{2} + 3\kappa_0 : 1 \quad (38)$$

with, to leading order in $|\lambda_u^s|T/|\lambda_t^s|P_{t-c}$ and P_{t-c}^{EW}/P_{t-c}

$$\begin{aligned} \kappa_\alpha = \cos \gamma \frac{|\lambda_u^s|T}{|\lambda_t^s|P_{t-c}} & \left(\cos(\delta_8 - \delta_{27}) \left(1 + \frac{C}{T} \right) + \alpha \left(1 - \frac{3C}{2T} \right) \right) \\ - \frac{P_{t-c}^{EW}}{P_{t-c}} & \left(\cos(\delta_8 - \delta_{27}) - \frac{\alpha}{4} \left(1 + \frac{5C}{C+T} \right) \right) \end{aligned} \quad (39)$$

For the range of parameter values in Table II, these corrections are of at most 15% (see Table VI).

To close this section, it should be noted that the SU(3)-elastic constraint $A_{\pi^0 K^+} = 2A_{\pi^- K^+}$, though consistent, is not favored by the present measurements of $A_{\pi^- K^+}$ by Belle, BaBar and $A_{\pi^0 K^+}$ by Belle:

	Belle	BaBar
$A_{\pi^- K^+}$	$-0.088 \pm 0.035 \pm 0.018[4]$	$-0.107 \pm 0.041 \pm 0.013[4]$
$A_{\pi^0 K^+}$	$0.23 \pm 0.11_{-0.04}^{+0.01}[2]$	$-0.09 \pm 0.09 \pm 0.01[3]$

Further, as shown in [12], this constraint is the same under SU(2)-elasticity. More precise measurements are necessary to draw any conclusion on the relevance of SU(N)-elastic rescatterings for $B \rightarrow PP$.

3.2 On-shell $c\bar{c}$ rescattering (no $PP \rightleftharpoons PP$)

For our second example, we take again the T and P_{t-c} amplitudes, but $\delta_8^D - \delta_8$ as the only strong phase. In addition, we have the T_D amplitude and the mixing angle χ . As a first step, since it is always the combination (χT_D) which appears in $B \rightarrow PP$ decays, we can use the measurements [22] $Br(B^0 \rightarrow D^- D_s^+) = (0.8 \pm 0.3)\%$ and $Br(B^+ \rightarrow \bar{D}^0 D_s^+) = (1.3 \pm 0.4)\%$ to fix $T_D = 2.43$ which gives

$$Br^{th}(B^0 \rightarrow D^- D_s^+) = Br^{th}(B^+ \rightarrow \bar{D}^0 D_s^+) = 0.98\% \quad (40)$$

For various weak angle γ we find

γ	60°	80°	100°
χ_{\min}^2	40	32	26
T	0.50	0.57	0.66
P_{t-c}	0.09	0.10	0.10
χ	3.3°	2.7°	2.6°
$\delta_8^D - \delta_8$	25°	21°	20°

Table III: T, P_{t-c} , on-shell $c\bar{c}$.

Note that the fitting procedure is delicate because, to a large extent, P_{t-c} and χT_D compete against each other (see Eq.(26)) and the χ_{\min}^2 function is rather flat. The above results are those for which both χ and $\delta_8^D - \delta_8$ are simultaneously small.

The $B \rightarrow \pi K$ branchings are now dominated by both P_{t-c} and $B \rightarrow \{\bar{D}D_s\} \rightarrow \pi K$ contributions, and since these effects behave like a "hadronic" penguin, see Eq.(26), the pattern Eq.(34) is not altered. Concerning CP-asymmetries, we find the pattern

$$A_{\pi^0 K^+} : A_{\pi^+ K^0} : A_{\pi^0 K^0} : A_{\pi^- K^+} \approx 1 : -\frac{P_{t-c}}{T} : -\frac{P_{t-c}}{T} : 1 \quad (41)$$

Interestingly, some ratios of CP-asymmetries directly give the value of P_{t-c}/T . These asymmetries are generated through the interference of T, P_{t-c} with $B \rightarrow \{\bar{D}D_s\} \rightarrow \pi K$, and require only one strong phase $\delta_8^D - \delta_8$ of less than about 25° (generated by the mismatch between $PP \rightleftharpoons PP$ and $DD \rightleftharpoons DD$, see Eq.(20)).

In the $B \rightarrow \pi\pi$ sector, the fit is not very good (the large χ_{\min}^2 is essentially generated by the $\pi^0\pi^0$ and $\pi^0\pi^+$ modes). Indeed, the $B \rightarrow \{\bar{D}D\} \rightarrow \pi\pi$ effects are Cabibbo-suppressed, and since there is no averaging from $PP \rightleftharpoons PP$ strong phases, the decay branchings keep their bare scalings in terms of dominant T (see appendix B)

$$Br(\pi^+\pi^0) : Br(\pi^+\pi^-) : Br(\pi^0\pi^0) \approx 1/2 : 1 : 0 \quad (42)$$

which is rather far from current measurements (but not yet ruled out, see table V, appendix C). Typically, $Br(B \rightarrow \pi^0\pi^0) < 10^{-6}$ without $PP \rightleftharpoons PP$ strong phases (and assuming $C/T \sim \mathcal{O}(\lambda)$ or smaller). Finally, $A_{\pi^+\pi^-}$ is found a bit smaller (in the range 0.2 to 0.4), while $S_{\pi^+\pi^-}$ behave roughly as in the SU(3)-elastic case (see Fig.5).

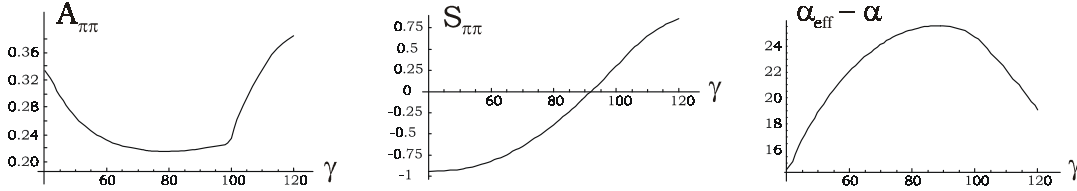


Figure 5: Fit result for $A_{\pi^+\pi^-}, S_{\pi^+\pi^-}$ and α_{eff} under on-shell $c\bar{c}$ rescattering.

Note, however, that $\alpha_{eff} \neq \alpha$ is now due to both $B \rightarrow \{\bar{D}D\} \rightarrow \pi\pi$ and P_{t-c} , and therefore that no P_{t-c} penguin does not imply $\alpha_{eff} = \alpha$ (this is obvious from Eq.(26)).

For $B \rightarrow K\bar{K}$, the branchings are similar than in the SU(3)-elastic case except for $B^+ \rightarrow K^+K^-$ which is exactly zero (no rescattering from $B \rightarrow \{\pi^+\pi^-\}$ and no $B \rightarrow \{\bar{D}D\} \rightarrow K^+K^-$ because of Eq.(30)). The $B^+ \rightarrow K^+\bar{K}^0$ and $B^0 \rightarrow K^0\bar{K}^0$ direct CP-asymmetries tend to be much smaller (between 0 and -0.3), and in addition verifies

$$A_{K^+\bar{K}^0} : A_{K^0\bar{K}^0} : A_{\pi^0\pi^0} = 1 : 1 : 1 \quad (43)$$

If we now include the subleading C and P_{t-c}^{EW} , the best fit procedure produces a large C amplitude to match the observed pattern of $B \rightarrow \pi\pi$ branchings (we have restricted

C/T to be at most 1/2)

γ	60°	80°	100°
χ_{\min}^2	18	14	12
T	0.49	0.55	0.62
C	0.24	0.20	0.23
P_{t-c}	0.09	0.09	0.09
P_{t-c}^{EW}	0.011	0.006	0.004
χ	3.5°	2.6°	4.7°
$\delta_8^D - \delta_8$	26°	26°	15°

Table IV: $T, P_{t-c}, C, P_{t-c}^{EW}$, on-shell $c\bar{c}$.

The pattern of $B \rightarrow \pi K$ direct CP-asymmetries is much affected by such a large C amplitude

$$A_{\pi^0 K^+} : A_{\pi^+ K^0} : A_{\pi^0 K^0} : A_{\pi^- K^+} = 1 + \frac{C}{T} + \frac{CP_{t-c}}{T^2} : -\frac{P_{t-c}}{T} : -\frac{P_{t-c} + C}{T} - \frac{CP_{t-c}}{T^2} : 1 \quad (44)$$

while corrections due to P_{t-c}^{EW} are subleading. Notice, however, that for the range of values in Table IV, this pattern is still different from the SU(3)-elastic one Eq.(35). Finally, the pattern Eq.(43) survive only partially: while $A_{K^+ \bar{K}^0}$ and $A_{K^0 \bar{K}^0}$ remain exactly equal (no C or P_{t-c}^{EW} contribution), $A_{\pi^0 \pi^0}$ is dominated by C and varies much.

To close this section, let us make a few comments. First, note that phenomenologically speaking, $P_{t-c}/T = 0$ is not yet ruled out by experiment once the penguin-like rescattering effects $B \rightarrow \{D\bar{D}\} \rightarrow PP$ are allowed. Indeed, the $B \rightarrow K\pi$ branchings could be dominated by the $B \rightarrow \{D_s \bar{D}\} \rightarrow \pi K$ contributions, while both $B \rightarrow \pi K$ and $B \rightarrow \pi^+ \pi^-$ direct CP-asymmetries could be generated by interference of $B \rightarrow \{D\bar{D}\} \rightarrow PP$ contributions with T . In other words, assuming that $D\bar{D} \Rightarrow PP$ is the dominant rescattering process, the current data requires $\left| P_{t-c}^{eff}/T \right| \neq 0$ with P_{t-c}^{eff} given in Eq.(26), but do not say anything yet about the relative strength of the quark-level penguin P_{t-c} vs. its hadronic-level counterpart $B \rightarrow \{D\bar{D}\} \rightarrow PP$. Now theoretically, from short-distance analyses there is good reasons to expect $P_{t-c}/T \neq 0$, and we hope that the phenomenological patterns of CP-asymmetries described above will help in extracting it from the data.

Second, under exact flavor SU(3), it appears that consistency with $B \rightarrow \pi K$ direct CP-asymmetry measurements restricts $A_{\pi\pi}$ to be smaller than ~ 0.5 . This statement is independent of the precise physics in the charmed meson sector. Except for the values quoted for χ , the results of the fits are equally valid when the intermediate charmed meson state is $D\bar{D}$, $D^* \bar{D}^*$, ... or a charmonium state like $P\eta_c$ (see [21]), or also combinations of them. Further, the rescatterings between these states is irrelevant for $B \rightarrow PP$. Note also that abandoning the coherence requirement Eq.(30) does not lead to larger $A_{\pi\pi}$, even if $PA_c^{eff} = PA_c + 2/3\beta + \alpha \approx 2/3\beta + \alpha$ (see Eq.(22)) contributes to $B^0 \rightarrow \pi^+ \pi^-$ and not to $B \rightarrow \pi K$, because PA_c^{eff} also contributes to $B^0 \rightarrow K^+ K^-$ and therefore cannot be large.

Finally, it should be noted that many relations between B^0, B^+ direct CP-asymmetry

and B_S ones exist

$$\begin{aligned} A_{B_S \rightarrow \pi^+ K^-} &= A_{B^0 \rightarrow \pi^+ \pi^-} & A_{B_S \rightarrow K^0 \bar{K}^0} &= A_{B^+ \rightarrow \pi^+ K^0} \\ A_{B_S \rightarrow \pi^0 \bar{K}^0} &= A_{B^0 \rightarrow \pi^0 \pi^0} & A_{B_S \rightarrow K^+ K^-} &= A_{B^0 \rightarrow \pi^- K^+} \end{aligned}$$

All these relations are unaffected by C or P_{t-c}^{EW} contributions and specific to the dominant on-shell $c\bar{c}$ rescattering source (no $PP \rightleftharpoons PP$ rescatterings).

4 Conclusions

The B to two charmless pseudoscalar decay processes are analyzed assuming exact SU(3) for the amplitudes and rescatterings. The basic tool is Watson's theorem for hadronic final state interactions, which allows for a factorization of rescattering effects from bare decay amplitudes. In addition, the various FSI processes can also be factorized and treated separately. In the present work, we assume SU(3)-elasticity for the $PP \rightleftharpoons PP$ part of the rescatterings, while $D\bar{D} \rightleftharpoons PP$ elastic rescatterings are shown to amount to the redefinition of the penguin amplitude as

$$\lambda_t P_{t-c}^{eff} = \lambda_t P_{t-c} - \lambda_c \sqrt{\frac{3}{5}} \left(1 - e^{i(\delta_8^D - \delta_8)}\right) \chi T_D \quad (45)$$

where T_D is the tree diagram contributing to $B \rightarrow D\bar{D}$ and χ the small mixing parameter governing the $D\bar{D}$ pollution of $B \rightarrow PP$ (this parameter is not fixed by SU(3), and must be small because of Eq.(6)). Importantly, the redefinition Eq.(45) also apply when the intermediate charmed meson state is $D^* \bar{D}^*$, $D^{**} \bar{D}^{**}$, $P\eta_c, \dots$, or any combination of them, and therefore Eq.(45) can be seen as a hadronic representation for on-shell intermediate $c\bar{c}$.

Also, it should be clear that once the various rescattering effects can be treated separately, different theoretical tools can be combined. This is well illustrated by Eq.(45): if one has a definite computation scheme for P_{t-c}^{eff} accounting for (elastic or inelastic) intermediate on-shell $c\bar{c}$, but suspects non-negligible elastic long-distance $PP \rightleftharpoons PP$ rescatterings, the present formalism permits the combination of the two. In other words, it is perfectly consistent to account for SU(N)-elastic long distance rescatterings by acting with the corresponding $\sqrt{S_{SU(N)}}$ on effective quark diagram amplitudes already containing various rescattering effects, like those produced by QCD-based approaches.

In our phenomenological approach, many amplitudes can be safely ignored and we are left with the set of free parameters Eq.(31), with, in addition, the weak angle γ . To further restrict the number of free parameters, one rescattering source is assumed dominant: either SU(3)-elasticity ($PP \rightleftharpoons PP$) or on-shell $c\bar{c}$ ($D\bar{D} \rightleftharpoons PP$). The most prominent features of each fit are:

SU(3)-elasticity: The $B \rightarrow \pi K$ direct CP-asymmetries tend to require large strong phases. This is nice for $B \rightarrow \pi\pi$ (see Eq.(36)) but may be problematic for $B \rightarrow KK$ (those modes are predicted close to their present experimental

upper bounds). Also, it should be noted that the SU(3)-elastic pattern of $B \rightarrow \pi K$ direct CP-asymmetries Eq.(35) is rather insensitive to subleading quark diagram amplitudes (C and P_{t-c}^{EW}), and is not in good agreement with present measurements (though the discrepancy is not yet significant).

On-shell $c\bar{c}$: A single strong phase difference ($\delta_8^D - \delta_8$) of around 25° is sufficient to produce the required asymmetries in the πK and $\pi\pi$ sectors. This strong phase difference is generated by the mismatch between $PP \rightleftharpoons PP$ and $D\bar{D} \rightleftharpoons D\bar{D}$ elastic rescattering channels. Such a small strong phase is nice from the point of view of Regge computations [15] and from the naive expectation for two particle flying apart with large momentum [16], but also with the SU(2)-elastic isospin analysis of $B \rightarrow D\pi$ [23]. Compared with the SU(3)-elastic case, the fit is not very good in the $B \rightarrow \pi\pi$ sector (no rescattering from $B \rightarrow \pi^+\pi^-$ leads to the approximate pattern Eq.(42)), but better for $B \rightarrow \pi K$ direct CP-asymmetries (the pattern Eq.(41) is favored by current measurements).

Combining both sources of rescattering would certainly improve the quality of the fit as more free parameters are available. For example, the difficulty of the on-shell $c\bar{c}$ fit in the $B \rightarrow \pi\pi$ sector is reduced by including a small amount of $PP \rightleftharpoons PP$ effects (like $\delta_{27} - \delta_8 \approx 20^\circ$). More precise measurements are necessary to pursue the analysis in that direction.

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References

- [1] CLEO Collaboration: S. Chen *et al.*, Phys. Rev. Lett. **85**, 525 (2000); D.M. Asner *et al.*, Phys. Rev. **D65**, 031103 (2002); A. Bornheim *et al.*, *hep-ex/0302026*.
- [2] BELLE Collaboration (K.Abe *et al.*): T. Tomura, presented at the 38th Rencontres de Moriond, March 22-29, 2003, Les Arcs, France, *hep-ex/0305036*; contributed paper to the XXI International Symposium on Lepton and Photon Interactions at High Energies, Fermilab Aug 11-16, 2003, *hep-ex/0308040*.
- [3] BABAR Collaboration (B. Aubert *et al.*), Phys. Rev. Lett. **89**, 281802 (2002); contributed to Flavor Physics and CP Violation (FPCP), Philadelphia, Pennsylvania, 16-18 May 2002, *hep-ex/0206053*; contributed to 31st International Conference on High Energy Physics, Amsterdam, The Netherlands, July 24-31, 2002, *hep-ex/0207065*; Phys. Rev. Lett. **91**, 021801 (2003); M. Bona, talk given at the 2003 Flavor Physics and CP Violation, Paris, June 3-6, 2003, BABAR-TALK-03065; J. Ocariz, presented

- at the International Europhysics Conference on HEP (EPS2003), Aachen, July 17-23, 2003, BABAR-TALK-03083; *hep-ex/0308012*.
- [4] John Fry, talk at the XXI International Symposium on Lepton and Photon Interactions at High Energies, Fermilab, Aug 11-16, 2003.
 - [5] BELLE Collaboration (K. Abe *et al.*), Phys. Rev. **D68**, 012001, (2003).
 - [6] BABAR Collaboration (B. Aubert *et al.*), preliminary result presented at the XXI International Symposium on Lepton and Photon Interactions at High Energies, Fermilab, Aug 11-16, 2003, BABAR-PLOT-0054.
 - [7] R. Fleischer and T. Mannel, Phys. Rev. **D57**, 2752 (1998); M. Gronau and J.L. Rosner, Phys. Rev. **D57**, 6843 (1998); R. Fleischer, Eur. Phys. J. **C6**, 451 (1999); Phys. Lett. **B435**, 221 (1998); M. Neubert and J.L. Rosner, Phys. Lett. **B441**, 403 (1998); Phys. Rev. Lett. **81**, 5076 (1998); M. Neubert, JHEP **9902**, 014 (1999); A.J. Buras and R. Fleischer, Eur. Phys. J. **C11**, 93 (1999); Eur. Phys. J. **C16**, 97 (2000); R. Fleischer and J. Matias, Phys. Rev. **D61**, 074004 (2000); J. Matias, Phys. Lett. **B520**, 131 (2001); M. Bargiotti *et al.*, Eur. Phys. J. **C24**, 361 (2002); M. Gronau and J.L. Rosner, Phys. Rev. **D65**, 013004 (2002); R. Fleischer and J. Matias, Phys. Rev. **D66**, 054009 (2002); A.I. Sanda and K. Ukai, Prog. Theor. Phys. **107**, 421 (2002); M. Neubert, talk given at International Workshop on Heavy Quarks and Leptons, Salerno, Italy, 27 May - 1 Jun 2002, *hep-ph/0207327*; Y.-Y. Keum, *hep-ph/0209208*; R. Fleischer, proceedings of International School on Heavy Quark Physics, Dubna, Russia, 27 May - 5 Jun 2002, *hep-ph/0210323*; Z.-J. Xiao, C.-D. Lu, L. Guo, *hep-ph/0303070*; M. Gronau and J. L. Rosner, *hep-ph/0307095*; M. Beneke and M. Neubert, *hep-ph/0308039*; Z.-Z. Xing, *hep-ph/0308225*; A. Buras, R. Fleischer, S. Recksiegel, F. Schwab, *hep-ph/0309012*.
 - [8] J. de Swart, Rev. Mod. Phys. **35**, 916 (1963); D. Zeppenfeld, Z. Phys. **C8**, 77 (1981); M. Savage and M. Wise, Phys. Rev. **D39**, 3346 (1989); L.-L. Chau, H.-Y. Cheng, W.K. Sze, H. Yao, B. Tseng, Phys. Rev. **D43**, 2176 (1991).
 - [9] M. Gronau, O. Hernandez, D. London, J. Rosner, Phys. Rev. **D50**, 4529 (1994); Phys. Rev. **D52**, 6356 (1995); Phys.Rev. **D52**, 6374 (1995).
 - [10] C. Smith, Eur. Phys. J. **C10**, 639 (1999).
 - [11] K. M. Watson, Phys. Rev. **88**, 1163 (1952).
 - [12] J.-M. Gérard and C. Smith, Eur. Phys. J. **C30**, 69 (2003).
 - [13] J.-M. Gérard and W.-S. Hou, Phys. Rev. Lett. **62**, 855 (1989); Phys. Rev. **D43**, 2909 (1991).
 - [14] W.-S. Hou, K.-C. Yang, Phys. Rev. Lett. **84**, 4806 (2000); C.-K. Chua, W.-S. Hou, K.-C. Yang, Mod. Phys. Lett. **A18**, 1763 (2003).

- [15] D. Delepine, J.-M. Gérard, J. Pestieau, J. Weyers, Phys. Lett. **B429**, 106 (1998); J.-M. Gérard, J. Pestieau, J. Weyers, Phys. Lett. **B436**, 363 (1998).
- [16] J.D. Bjorken, Nucl. Phys. Proc. Suppl. **11**, 325 (1989); M.J. Duncan and B. Grinstein, Phys. Lett. **B255**, 583 (1991); H.D. Politzer and M.B. Wise, Phys. Lett. **B257**, 399 (1991).
- [17] M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Nucl. Phys. **B606**, 245 (2001).
- [18] G. 't Hooft, Nucl. Phys. **B72**, 461 (1974); A.J. Buras, J.-M. Gérard, R. Ruckl, Nucl. Phys. **B268**, 16 (1986).
- [19] J.-M. Gérard and J. Weyers, Eur. Phys. J. **C7**, 1 (1999); M. Neubert, Phys. Lett. **B424**, 152 (1998).
- [20] Previous attempts to include $\bar{D}D$ ($\bar{D}D_s$) contributions to $B \rightarrow \pi\pi$ (πK) can be found in: L.M. Sehgal and M. Wanninger, Z. Phys. **C50**, 47 (1991); A. N. Kamal, Phys. Rev. **D60**, 094018 (1999); Z. Xing, Phys. Lett. **B493**, 301 (2000).
- [21] S. Barshay and G. Kreyerhoff, JHEP **0309**, 004 (2003).
- [22] Particle Data Group Collaboration: K. Hagiwara *et al.*, Phys. Rev. **D66**, 010001 (2002).
- [23] CLEO collaboration: S. Ahmed *et al.*, Phys. Rev. **D66**, 031101 (2002).
- [24] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).
- [25] BABAR Collaboration (B. Aubert *et al.*), Phys. Rev. Lett. **89**, 201802 (2002); BELLE Collaboration (K. Abe *et al.*), contributed to the XXI International Symposium on Lepton and Photon Interactions at High Energies, Fermilab, Aug. 11-16, 2003, *hep-ph/0308036*.

B Decay Amplitude Expressions

The $B \rightarrow PP$ decay amplitudes, under the approximations discussed in the text, are

$$\begin{aligned}
A(B^+ \rightarrow K^+ \bar{K}^0) &= \frac{e^{i\delta_8} - e^{i\delta_{27}}}{5} (T + C) + e^{i\delta_8} P_{t-c}^{eff} \\
A(B^0 \rightarrow K^+ K^-) &= \frac{5(1 - e^{i\delta_8}) + (e^{i\delta_8} - e^{i\delta_{27}})}{20} T - \frac{5(1 - e^{i\delta_8}) - 3(e^{i\delta_8} - e^{i\delta_{27}})}{60} C + \frac{2(1 - e^{i\delta_8})}{3} P_{t-c}^{eff} \\
A(B^0 \rightarrow K^0 \bar{K}^0) &= \frac{5(1 - e^{i\delta_8}) + (e^{i\delta_8} - e^{i\delta_{27}})}{20} T - \frac{5(1 - e^{i\delta_8}) - 3(e^{i\delta_8} - e^{i\delta_{27}})}{60} C + \frac{2 + e^{i\delta_8}}{3} P_{t-c}^{eff} \\
A(B^+ \rightarrow \pi^+ \pi^0) &= \frac{e^{i\delta_{27}}}{\sqrt{2}} (T + C) \\
A(B^0 \rightarrow \pi^+ \pi^-) &= \frac{5 + 8e^{i\delta_8} + 7e^{i\delta_{27}}}{20} T - \frac{5(1 - e^{i\delta_8}) + 21(e^{i\delta_8} - e^{i\delta_{27}})}{60} C + \frac{2 + e^{i\delta_8}}{3} P_{t-c}^{eff} \\
A(B^0 \rightarrow \pi^0 \pi^0) &= \frac{5(1 - e^{i\delta_8}) + 13(e^{i\delta_8} - e^{i\delta_{27}})}{20\sqrt{2}} T - \frac{5 + 16e^{i\delta_8} + 39e^{i\delta_{27}}}{60\sqrt{2}} C + \frac{2 + e^{i\delta_8}}{3\sqrt{2}} P_{t-c}^{eff} \\
A(B^+ \rightarrow K^+ \pi^0) &= \frac{e^{i\delta_8} + 4e^{i\delta_{27}}}{5\sqrt{2}} (T + C) + \frac{e^{i\delta_8}}{\sqrt{2}} P_{t-c}^{eff} \\
A(B^+ \rightarrow K^0 \pi^+) &= \frac{e^{i\delta_8} - e^{i\delta_{27}}}{5} (T + C) + e^{i\delta_8} P_{t-c}^{eff} \\
A(B^0 \rightarrow K^+ \pi^-) &= \frac{3e^{i\delta_8} + 2e^{i\delta_{27}}}{5} T + \frac{2(e^{i\delta_{27}} - e^{i\delta_8})}{5} C + e^{i\delta_8} P_{t-c}^{eff} \\
A(B^0 \rightarrow K^0 \pi^0) &= \frac{3(e^{i\delta_{27}} - e^{i\delta_8})}{5\sqrt{2}} T + \frac{2e^{i\delta_8} + 3e^{i\delta_{27}}}{5\sqrt{2}} C - \frac{e^{i\delta_8}}{\sqrt{2}} P_{t-c}^{eff} \\
A(B^+ \rightarrow \pi^+ \eta_8) &= \frac{2e^{i\delta_8} + 3e^{i\delta_{27}}}{5\sqrt{6}} (T + C) + \frac{2e^{i\delta_8}}{\sqrt{6}} P_{t-c}^{eff} \\
A(B^0 \rightarrow \pi^0 \eta_8) &= -\frac{e^{i\delta_8}}{\sqrt{3}} P_{t-c}^{eff} \\
A(B^+ \rightarrow K^+ \eta_8) &= -\frac{e^{i\delta_8} - 6e^{i\delta_{27}}}{5\sqrt{6}} (T + C) - \frac{e^{i\delta_8}}{\sqrt{6}} P_{t-c}^{eff} \\
A(B^0 \rightarrow K^0 \eta_8) &= \frac{3(e^{i\delta_{27}} - e^{i\delta_8})}{5\sqrt{6}} T + \frac{2e^{i\delta_8} + 3e^{i\delta_{27}}}{5\sqrt{6}} C - \frac{e^{i\delta_8}}{\sqrt{6}} P_{t-c}^{eff} \\
A(B^0 \rightarrow \eta_8 \eta_8) &= \frac{5(1 - e^{i\delta_8}) - 3(e^{i\delta_8} - e^{i\delta_{27}})}{20\sqrt{2}} T - \frac{5 - 16e^{i\delta_8} - 9e^{i\delta_{27}}}{60\sqrt{2}} C + \frac{2 - e^{i\delta_8}}{3\sqrt{2}} P_{t-c}^{eff}
\end{aligned}$$

The amplitudes for the charge-conjugate modes are identical, except for the understood CKM coefficients. Decay amplitudes for B_S mode are:

$$\begin{aligned}
A(B_S \rightarrow K^- \pi^+) &= \frac{3e^{i\delta_8} + 2e^{i\delta_{27}}}{5} T - \frac{2(e^{i\delta_8} - e^{i\delta_{27}})}{5} C + e^{i\delta_8} P_{t-c}^{eff} \\
A(B_S \rightarrow \bar{K}^0 \pi^0) &= -\frac{3(e^{i\delta_8} - e^{i\delta_{27}})}{5\sqrt{2}} T + \frac{2e^{i\delta_8} + 3e^{i\delta_{27}}}{5\sqrt{2}} C - \frac{e^{i\delta_8}}{\sqrt{2}} P_{t-c}^{eff} \\
A(B_S \rightarrow \bar{K}^0 \eta_8) &= -\frac{3(e^{i\delta_8} - e^{i\delta_{27}})}{5\sqrt{6}} T + \frac{2e^{i\delta_8} + 3e^{i\delta_{27}}}{5\sqrt{6}} C - \frac{e^{i\delta_8}}{\sqrt{6}} P_{t-c}^{eff} \\
A(B_S \rightarrow K^+ K^-) &= \frac{5 + 8e^{i\delta_8} + 7e^{i\delta_{27}}}{20} T - \frac{5(1 - e^{i\delta_8}) + 21(e^{i\delta_8} - e^{i\delta_{27}})}{60} C + \frac{2 + e^{i\delta_8}}{3} P_{t-c}^{eff} \\
A(B_S \rightarrow K^0 \bar{K}^0) &= \frac{5(1 - e^{i\delta_8}) + (e^{i\delta_8} - e^{i\delta_{27}})}{20} T - \frac{5(1 - e^{i\delta_8}) - 3(e^{i\delta_8} - e^{i\delta_{27}})}{60} C + \frac{2 + e^{i\delta_8}}{3} P_{t-c}^{eff} \\
A(B_S \rightarrow \pi^+ \pi^-) &= \frac{5(1 - e^{i\delta_8}) + (e^{i\delta_8} - e^{i\delta_{27}})}{20} T - \frac{5(1 - e^{i\delta_8}) - 3(e^{i\delta_8} - e^{i\delta_{27}})}{60} C + \frac{2(1 - e^{i\delta_8})}{3} P_{t-c}^{eff} \\
A(B_S \rightarrow \pi^0 \pi^0) &= \frac{5(1 - e^{i\delta_8}) + (e^{i\delta_8} - e^{i\delta_{27}})}{20\sqrt{2}} T - \frac{5(1 - e^{i\delta_8}) - 3(e^{i\delta_8} - e^{i\delta_{27}})}{60\sqrt{2}} C + \frac{2(1 - e^{i\delta_8})}{3\sqrt{2}} P_{t-c}^{eff} \\
A(B_S \rightarrow \pi^0 \eta_8) &= \frac{3(e^{i\delta_8} - e^{i\delta_{27}})}{5\sqrt{3}} T - \frac{2e^{i\delta_8} + 3e^{i\delta_{27}}}{5\sqrt{3}} C \\
A(B_S \rightarrow \eta_8 \eta_8) &= \frac{5(1 - e^{i\delta_8}) + 9(e^{i\delta_8} - e^{i\delta_{27}})}{20\sqrt{2}} T - \frac{5 + 8e^{i\delta_8} + 27e^{i\delta_{27}}}{60\sqrt{2}} C + \frac{2(1 + e^{i\delta_8})}{3\sqrt{2}} P_{t-c}^{eff}
\end{aligned}$$

Finally, assuming negligible $PP, D\bar{D} \rightleftharpoons D^*\bar{D}^*, D^{**}\bar{D}^{**}, \dots$, the $B \rightarrow D\bar{D}$ decay amplitudes are

$$\begin{aligned}
\mathcal{A}(B^+ \rightarrow D_s^+ \bar{D}^0) &= \mathcal{A}(B^0 \rightarrow D_s^+ D^-) = e^{i\delta_8^D} T_D \\
\mathcal{A}(B^+ \rightarrow D^+ \bar{D}^0) &= \mathcal{A}(B_S \rightarrow D^+ D_s^-) = e^{i\delta_8^D} T_D \\
\mathcal{A}(B^0 \rightarrow D^+ D^-) &= \frac{1}{3} e^{i\delta_8^D} (2 + e^{-i\delta_8}) T_D \\
\mathcal{A}(B^0 \rightarrow D^0 \bar{D}^0) &= -\mathcal{A}(B^0 \rightarrow D_s^+ D_s^-) = \frac{1}{3} e^{i\delta_8^D} (1 - e^{-i\delta_8}) T_D \\
\mathcal{A}(B_S^0 \rightarrow D_s^+ D_s^-) &= \frac{1}{3} e^{i\delta_8^D} (2 + e^{-i\delta_8}) T_D \\
\mathcal{A}(B_S \rightarrow D^0 \bar{D}^0) &= -\mathcal{A}(B_S \rightarrow D^+ D^-) = \frac{1}{3} e^{i\delta_8^D} (1 - e^{-i\delta_8}) T_D
\end{aligned}$$

C Fit Results for Branchings and Asymmetries

The experimental data on B to two pseudoscalar decays we shall use are summarized in table V and VI, where direct CP-asymmetries are defined according to the sign convention

$$A_{CP} = \frac{\Gamma(\bar{B} \rightarrow \bar{f}) - \Gamma(B \rightarrow f)}{\Gamma(\bar{B} \rightarrow \bar{f}) + \Gamma(B \rightarrow f)} \quad (46)$$

Average is made over CLEO [1], Belle [2] and BaBar [3] measurements (see also [4]), assuming no correlation. The implications of SU(3) symmetry bear on the decay amplitudes, so predictions for branchings must be corrected to account for lifetime differences and available phase-space. From [22], the lifetime correction factor is $\Gamma_{tot}(B^+)/\Gamma_{tot}(B^0) = 0.92$, while that of phase space are of at most a few percents, and are neglected. The branchings under brackets in table V are branching fractions for the physical η state and not for η_8 , so that they are only indicative. Indeed, the small admixture of singlet η_0 into the physical η state could lead to large effects. The last observables of interest to us are the time-dependent asymmetry parameters

$$A_{f\bar{f}} = \frac{|\lambda_{f\bar{f}}|^2 - 1}{|\lambda_{f\bar{f}}|^2 + 1} = A_{CP}, \quad S_{f\bar{f}} = \frac{2 \text{Im} \lambda_{f\bar{f}}}{|\lambda_{f\bar{f}}|^2 + 1}, \quad \text{with } \lambda_{f\bar{f}} = e^{-2i\beta} \frac{A(\bar{B} \rightarrow f\bar{f})}{A(B \rightarrow f\bar{f})} \quad (47)$$

The experimental situation and averages are (with inflated errors, see [22]):

	Belle [5]	BaBar [6]	Average
$A_{\pi^+\pi^-}$	$0.77 \pm 0.27 \pm 0.08$	$0.19 \pm 0.19 \pm 0.05$	0.38 ± 0.27
$S_{\pi^+\pi^-}$	$-1.23 \pm 0.41 \pm 0.08$	$-0.40 \pm 0.22 \pm 0.03$	-0.58 ± 0.34

We have used only $A_{\pi^+\pi^-} = -C_{\pi^+\pi^-}$ as input.

The function we minimize to find the best-fit values is as usual

$$\chi^2 = \sum_f \left(\frac{Br^{th} - Br^{\text{exp}}}{\sigma_{Br}^{\text{exp}}} \right)^2 + \sum_f \left(\frac{A_{CP}^{th} - A_{CP}^{\text{exp}}}{\sigma_{A_{CP}}^{\text{exp}}} \right)^2 \quad (48)$$

where the sum runs over measured decay branchings and asymmetries (12 inputs). Upper bounds are implemented using the arc-tangent representation of the step function (3 inputs).

Finally, the Wolfenstein parametrization [24] of the CKM matrix elements is used

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} - \left(\frac{A^2}{2} + \frac{1}{8}\right)\lambda^4 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 + A\lambda^4\left(\frac{1}{2} - \rho - i\eta\right) & 1 - \frac{A^2\lambda^4}{2} \end{pmatrix} \quad (49)$$

and we take the central values of [22]

$$\lambda = 0.2196 \pm 0.0023, \quad A = 0.854 \pm 0.045, \quad \sqrt{\rho^2 + \eta^2} = 0.43 \pm 0.07 \quad (50)$$

and keep only the weak angle $\gamma = \arctan \eta/\rho$ as a free parameter (so unitarity of V_{CKM} is implied). For the parameter $S_{f\bar{f}}$, we use the average $\sin 2\beta = 0.736 \pm 0.049$ extracted from charmonium modes [25].

The result for branchings and asymmetries for the fits are collected below. Column labels refer to the best-fit parameter tables given in the text.

Branching ($\times 10^{-6}$)			SU(3)-elastic				On-shell $c\bar{c}$			
			I		II		III		IV	
$\Delta S = 0$	Exp.		60°	90°	60°	90°	60°	90°	60°	90°
B_S	$\overline{K^0}\eta_8$	—	0.43	0.37	0.31	0.42	0.13	0.18	0.22	0.45
	$\overline{K^0}\pi^0$	—	1.29	1.10	0.93	1.26	0.38	0.53	0.65	1.35
	$K^-\pi^+$	—	5.91	5.78	5.72	4.61	4.77	4.72	4.55	4.24
B^+	$\pi^+\eta_8$	[4.0±0.9]	1.59	1.38	2.41	1.72	1.42	1.17	2.31	1.93
	$\overline{K^0}K^+$	<1.3	1.09	1.28	1.01	1.28	0.88	1.21	0.89	1.18
	$\pi^+\pi^0$	5.3±0.8	3.48	4.42	5.43	5.67	2.07	3.10	4.34	5.86
B^0	K^+K^-	<0.6	0.60	0.60	0.60	0.15	0	0	0	0
	$K^0\overline{K^0}$	<1.6	1.02	1.31	0.93	1.14	0.81	1.11	0.82	1.08
	$\eta_8\eta_8$	—	0.17	0.25	0.12	0.09	0.05	0.06	0.11	0.09
	$\pi^0\eta_8$	[<2.9]	0.26	0.38	0.26	0.38	0.27	0.37	0.27	0.36
	$\pi^+\pi^-$	4.6±0.4	4.82	4.79	4.58	4.53	5.03	4.98	4.80	4.47
	$\pi^0\pi^0$	1.92±0.44	2.12	1.70	1.77	1.57	0.41	0.56	0.68	1.43
$\Delta S = 1$	Exp.		60°	90°	60°	90°	60°	90°	60°	90°
B^0	$K^0\eta_8$	[2.4±0.9]	3.33	3.17	2.94	2.82	3.37	3.17	2.92	2.86
	$K^0\pi^0$	11.2±1.4	9.98	9.51	8.81	8.45	10.1	9.51	8.75	8.57
	$K^+\pi^-$	18.2±0.8	18.6	19.2	18.3	19.0	18.6	19.2	18.5	19.1
B^+	$K^+\eta_8$	[3.1±0.7]	4.02	3.52	3.68	3.20	4.02	3.51	3.50	3.20
	$K^+\pi^0$	12.8±1.1	10.3	10.4	11.7	11.7	10.1	10.4	11.6	11.6
	$K^0\pi^+$	20.6±1.4	22.0	20.6	22.3	20.7	22.0	20.6	21.8	20.5
B_S	K^+K^-	—	15.5	14.7	14.2	16.9	17.6	18.2	17.6	18.1
	$K^0\overline{K^0}$	—	16.6	14.5	15.9	17.0	19.2	18.0	19.0	17.9
	$\eta_8\eta_8$	—	14.1	12.5	12.9	14.2	17.0	16.0	15.8	15.2
	$\pi^0\eta_8$	—	0.03	0.02	0.05	0.04	0	0	0.05	0.03
	$\pi^+\pi^-$	—	5.19	6.95	6.55	1.95	0	0	0	0
	$\pi^0\pi^0$	—	2.59	3.48	3.28	0.97	0	0	0	0

Table V: Fit results for charge-average branching fractions.

Direct CP-asymmetry A_{CP} (%)			SU(3)-elastic				On-shell $c\bar{c}$			
			I		II		III		IV	
$\Delta S = 0$	Exp.		60°	90°	60°	90°	60°	90°	60°	90°
B_S	$\overline{K^0}\eta_8$	—	-78	-99	-88	-70	-32	-19	-72	-36
	$\overline{K^0}\pi^0$	—	-78	-99	-88	-70	-32	-19	-72	-36
	$K^-\pi^+$	—	23	25	19	26	23	22	26	31
B^+	$\pi^+\eta_8$	—	49	60	28	41	23	27	27	35
	$\overline{K^0}K^+$	—	-71	-64	-67	-56	-32	-19	-33	-24
	$\pi^+\pi^0$	-7 ± 15	0	0	0	0	0	0	0	0
B^0	K^+K^-	—	14	16	11	18	-	-	-	-
	$K^0\overline{K^0}$	—	-88	-95	-87	-58	-32	-19	-33	-24
	$\eta_8\eta_8$	—	-96	-98	-16	39	-32	-19	22	33
	$\pi^0\eta_8$	—	0	0	0	0	-32	-19	-33	-24
	$\pi^+\pi^-$	38 ± 27	42	54	43	42	23	22	26	31
	$\pi^0\pi^0$	—	-49	-70	-69	-85	-32	-19	-72	-36
$\Delta S = 1$	Exp.		60°	90°	60°	90°	60°	90°	60°	90°
B^0	$K^0\eta_8$	—	10	12	9.5	11	1.2	1.1	5.5	5.9
	$K^0\pi^0$	3 ± 37	10	12	9.5	11	1.2	1.1	5.5	5.9
	$K^+\pi^-$	-9.3 ± 2.9	-7.5	-7.7	-6.1	-6.4	-6.1	-5.5	-6.7	-7.0
B^+	$K^+\eta_8$	—	19	23	18	22	7.4	7.6	13	15
	$K^+\pi^0$	1 ± 12	-15	-15	-11	-12	-6.1	-5.5	-9.1	-9.8
	$K^0\pi^+$	1 ± 6	3.4	3.9	2.9	3.4	1.2	1.1	1.3	1.3
B_S	K^+K^-	—	-12	-16	-13	-11	-6.1	-5.5	-6.7	-7.0
	$K^0\overline{K^0}$	—	5.0	7.9	4.7	3.6	1.2	1.1	1.3	1.3
	$\eta_8\eta_8$	—	8.1	11	9.5	8.7	1.2	1.1	3.3	3.5
	$\pi^0\eta_8$	—	0	0	-80	-71	-	-	0	0
	$\pi^+\pi^-$	—	-1.4	-1.3	-0.9	-1.3	-	-	-	-
	$\pi^0\pi^0$	—	-1.4	-1.3	-0.9	-1.3	-	-	-	-

Table VI: Fit results for direct CP-asymmetries (A_{CP}).

$S_{f\bar{f}}$ (%)			SU(3)-elastic				On-shell $c\bar{c}$			
			I		II		III		IV	
$\Delta S = 0$	Exp.		60°	90°	60°	90°	60°	90°	60°	90°
B^0	K^+K^-	—	-99	-83	-99	-90	-	-	-	-
	$K^0\overline{K^0}$	—	-30	2.3	-27	-5.2	-19	-10	-24	-16
	$\eta_8\eta_8$	—	-23	-18	-49	-36	-19	-10	-97	-86
	$\pi^0\eta_8$	—	6.2	-1.2	6.2	-1.2	-19	-10	-24	-16
	$\pi^+\pi^-$	-58 ± 34	-69	16	-67	-1.3	-81	-6.5	-81	-9.7
	$\pi^0\pi^0$	—	-67	8.4	-39	51	-19	-10	65	86

Table VII: Fit results for the time-dependent CP-asymmetry parameter $S_{f\bar{f}}$.